Math 10B with Professor Stankova
Worksheet, Discussion #7; Thursday, 2/8/2018
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1 Induction

1.1 Examples

1. Prove that $5^{2n+1} + 2^{2n+1}$ is divisible by 7 for all $n \geq 0$.
2. Find a formula for
$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \cdots + \frac{1}{(2n-1)(2n+1)}$$
for $n \geq 1$

1.2 Problems

3. True False If we want to prove S_n for all $n \geq 10$, then our base case would be $n = 10$.
4. True False When using induction, if we can show that if S_{100} is true, then S_{101} is true, then S_n must be true for all n.
5. True False Instead of assuming S_n is true and showing that S_{n+1} is true, we can instead assume that S_{n-1} is true and prove that S_n is true.
6. Prove that $n^3 + 2n$ is divisible by 3 for all integers $n \geq 0$.
7. Find a formula for
$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \cdots + \frac{1}{n(n+1)}.$$
8. Prove that for $n \geq 1$
$$1 + 4 + 7 + \cdots + (3n-2) = \frac{n(3n-1)}{2}.$$
9. Prove that $6^n - 1$ is divisible by 5 for $n \geq 1$.
10. Prove that $1! \cdot 1 + 2! \cdot 2 + 3! \cdot 3 + \cdots + n! \cdot n = (n+1)! - 1$. for all $n \geq 1$.
11. Prove that $1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + n \times (n+1) = \frac{n(n+1)(n+2)}{3}$ for all $n \geq 1$.
12. (Challenge) Let the Fibonacci numbers be defined as $F_1 = F_2 = 1$ and $F_{n+2} = F_n + F_{n+1}$. Then show that
$$F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$$

2 Probability

2.1 Examples

13. Assume that a telephone number is a 7 digit number that does not begin with 0 or 1. If I pick a random telephone number, what is the probability that it begins with a 9 or ends with a 0? What is the probability space?

14. In the 3 classes I'm taking, each has 3 HW assignments that have to be done in order (for a total of 9 HW assignments). If I randomly pick an order to do these 9 HWs, what is the probability that I actually do each of the 3 HWs in order?

2.2 Problems

15. True False The probability function P takes outcomes and outputs a probability for that outcome.

16. True False When calculating the probability of an event $A \subset \Omega$, we can always take $P(A) = |A|/|\Omega|$.

17. I roll 4 6-sided die. What is the probability that the sum of the numbers rolled is 7?

18. I am giving out grades to 60 students randomly (A, B, C, D, F). What is the probability that at least half the class got A's?

19. Out of the 14 pants that I own, there are 5 of them that are white. Every day for two weeks, I randomly put on a new pair of pants. What is the probability that I won't wear white pants two days in a row?

20. I pick a random 5 digit number. What is the probability that its digits are in increasing (could be repeating) order?

21. I roll 5 die for Yahtzee. What is the probability that I get a 5 in a row?

22. I pick 3 numbers a, b, c (not necessarily different) numbers from $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. What is the probability that $ab + c$ is even?